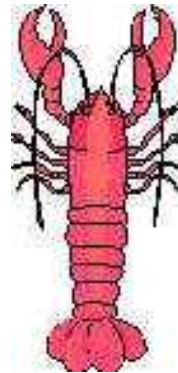


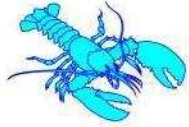
## **Schedule Human Resources in Retail organizations**

### **A branch and price framework using mixed OR techniques**



David GRAVOT  
Nabil GUERINIK  
Malika HADJIAT

ROSTUDEL Operations Research Software and Consulting  
[www.rostudel.com](http://www.rostudel.com)



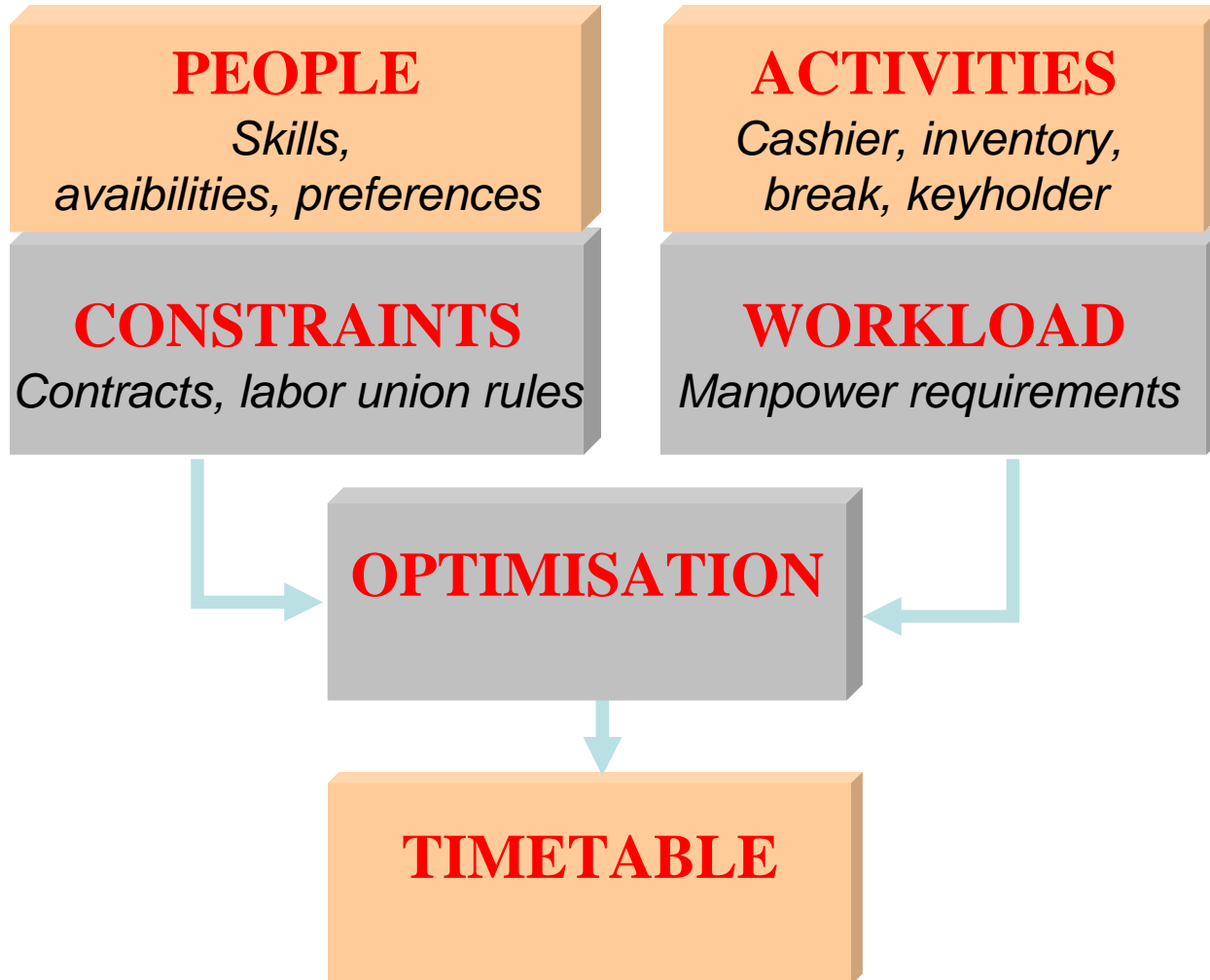
# outline

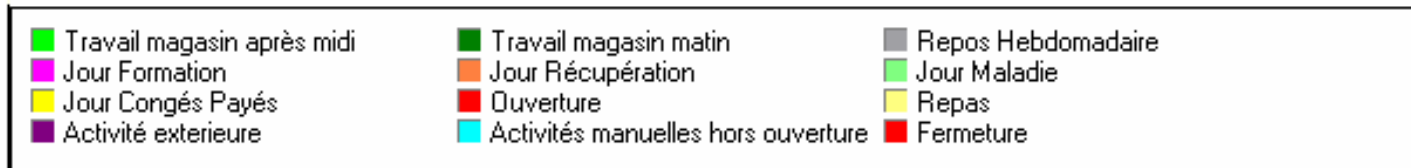
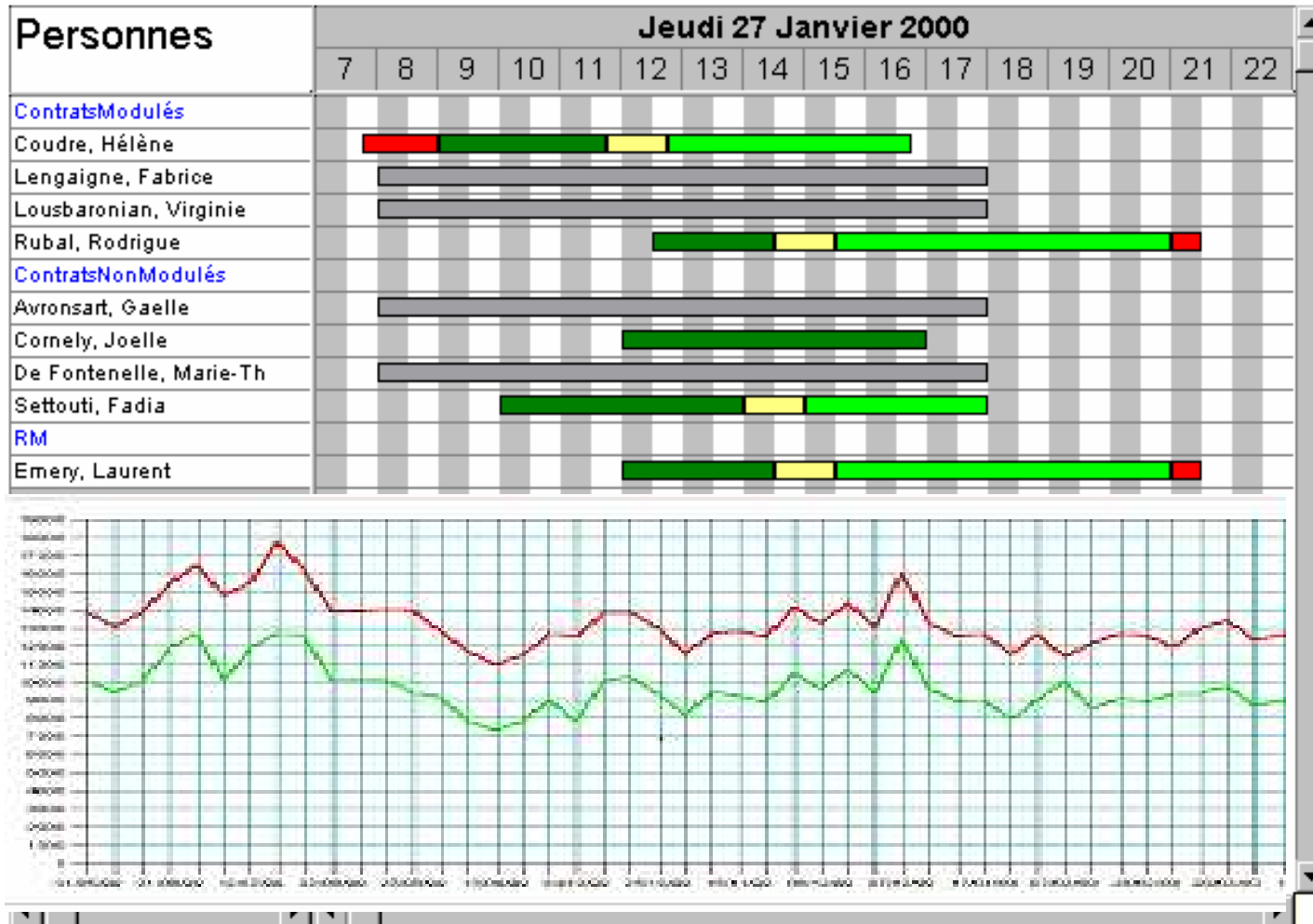
**Scope**

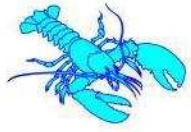
**Modelization => Towards decomposition**

**Solving => Mixed OR techniques**

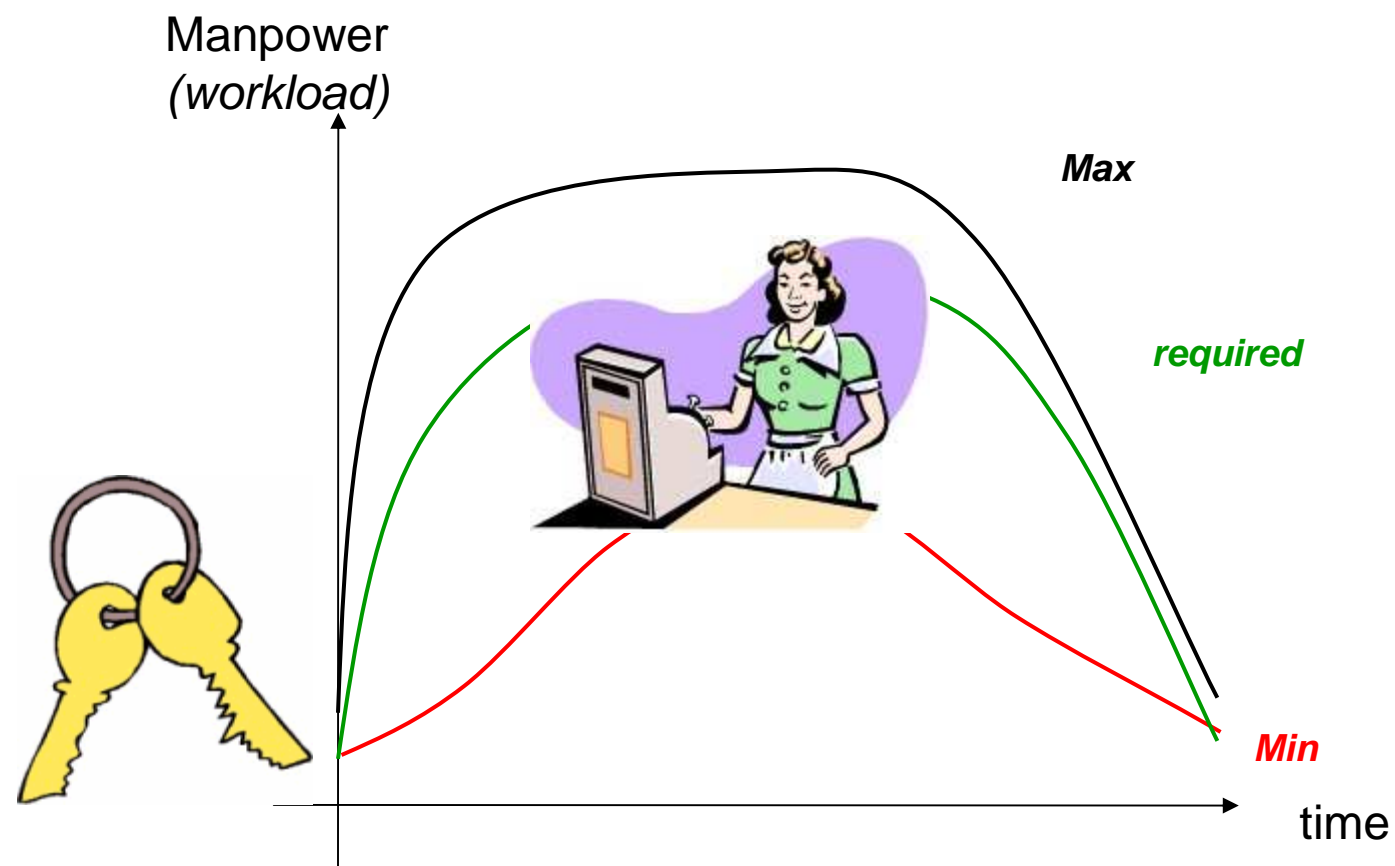
**Conclusion**







# Activities



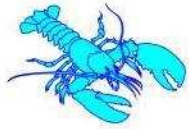
**Unbreakable**



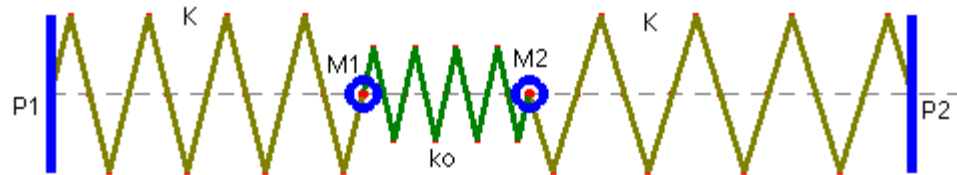
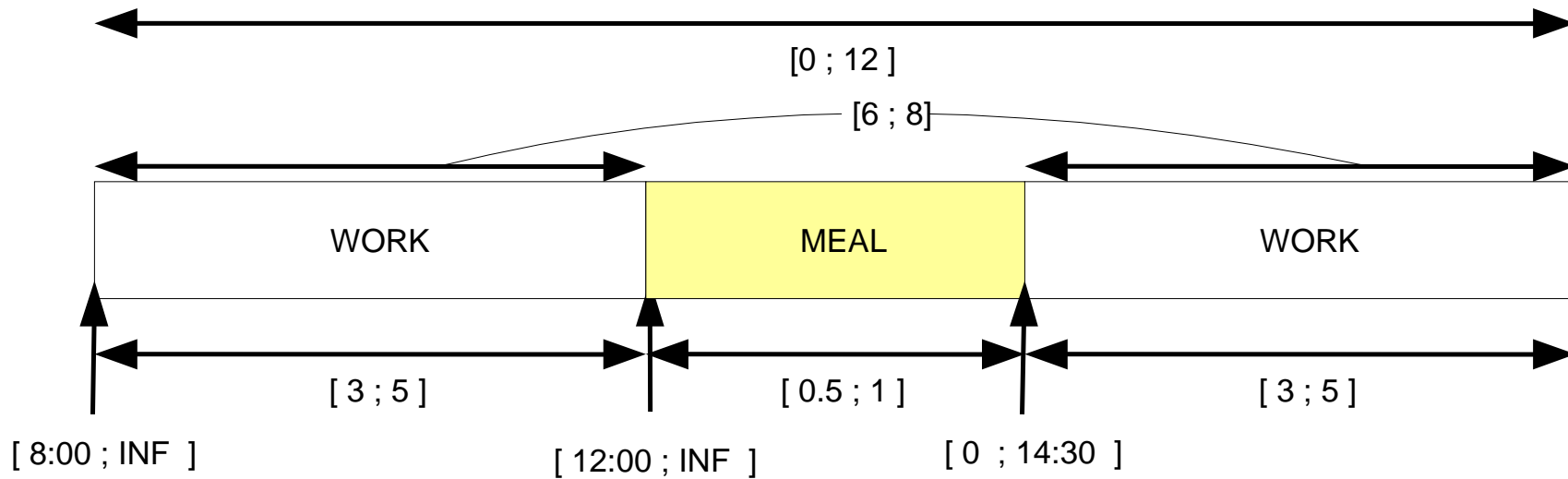
**Mobile**



**Mobile**



# People have a choice of Day patterns



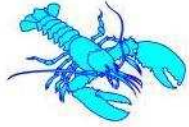
Two Breaks  
Day Off  
Mid Day...



## Constraints beyond day

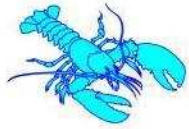
- Min/Max weekly work duration
- Min/Max working (rest) days
- Min/Max consecutive working (rest) days
- Min rest time between two consecutive working days
- Number of "late night" working days
- ...

**User ruled : not known in advance !**



## Objectives

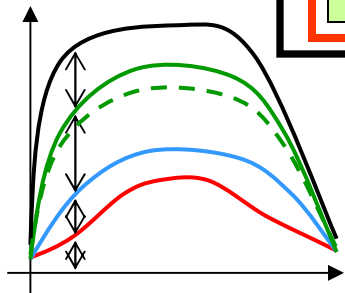
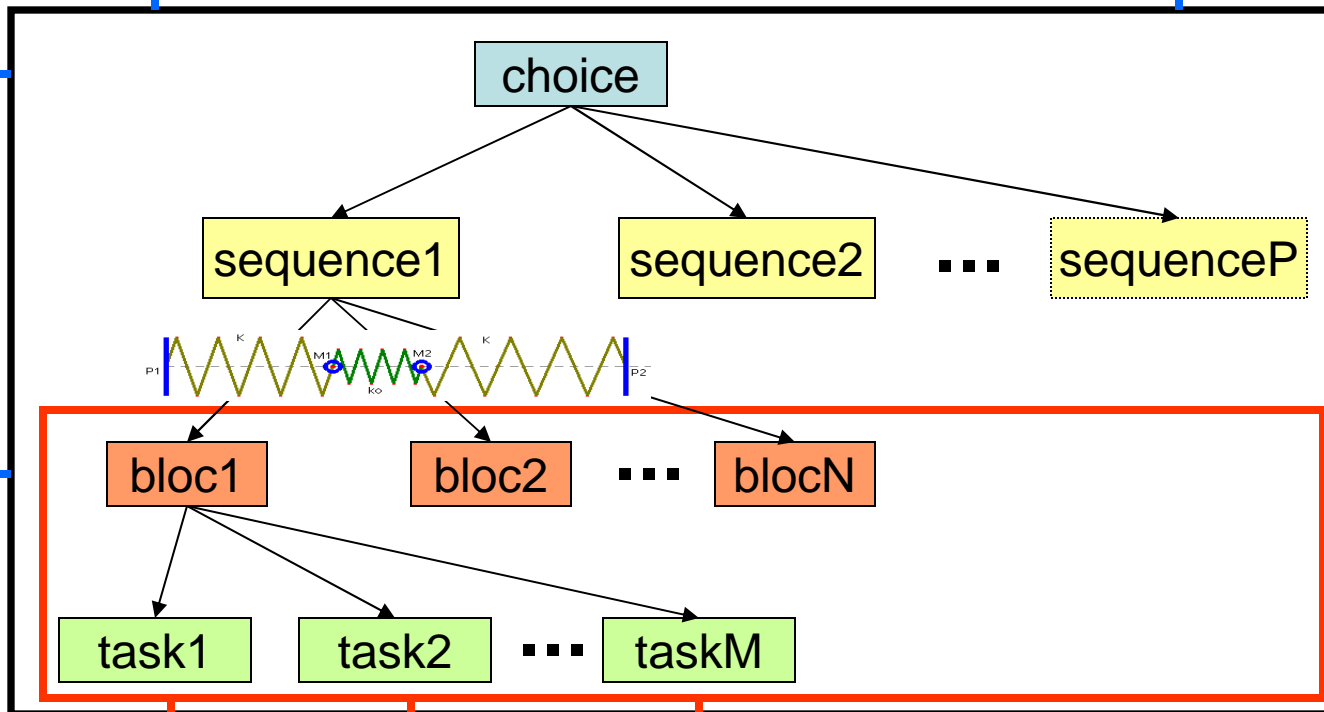
- Soft constraints slacks
- Under and over load
- Schedule cost
- Preferences violations, skill priorities
- Bound/Minimize number of activities transitions within a work shift



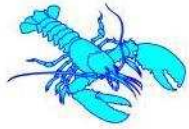
# Day as a "combinatorial" frontier



ROW CONSTRAINTS use employee aggregated variables  
Boolean, Begin, End, Management duration, work duration...



COLUMN CONSTRAINTS use task accurate position  
15 minutes scope, concerning a group of people



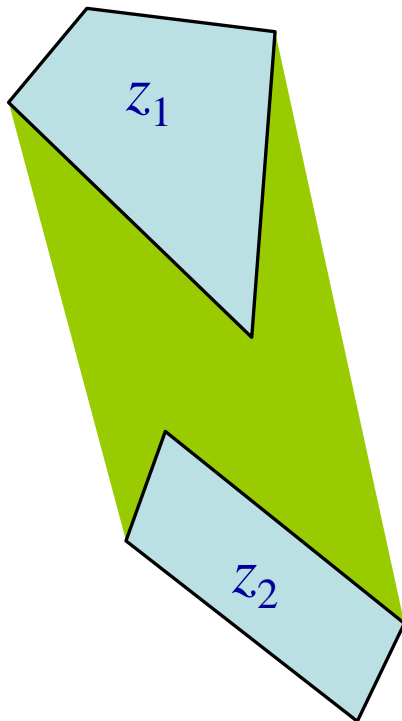
# DISJUNCTIVE PROGRAMMING

Courtesy of P. Refalo  
(ILOG) – ROADEF2003

Given a disjunction

$$A^1x \leq b^1 \vee A^2x \leq b^2 \vee \dots \vee A^nx \leq b^n$$

The following system represents  
its convex hull [Balas 75]



$$A^1x^1 \leq b^1z_1$$

$$A^2x^2 \leq b^2z_2$$

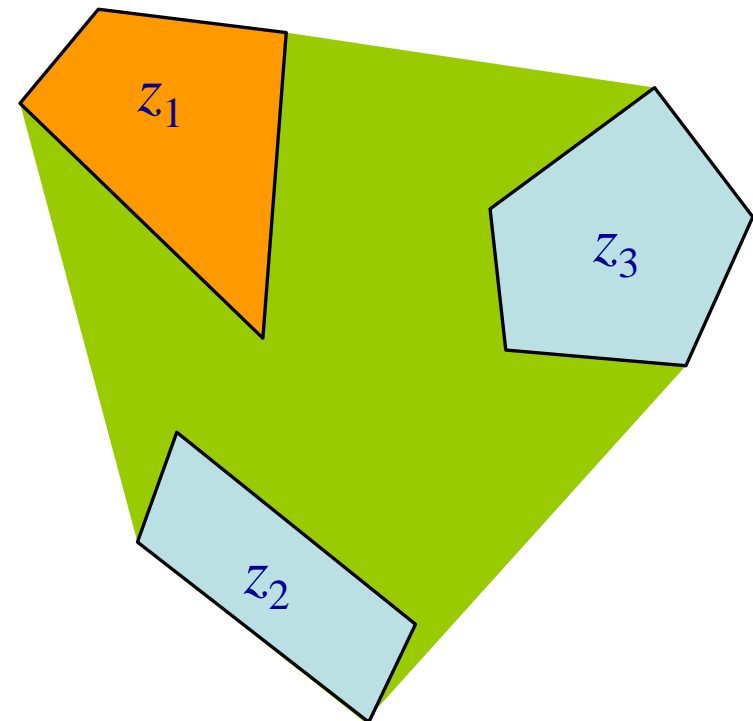
...

$$A^nx^n \leq b^nz_n$$

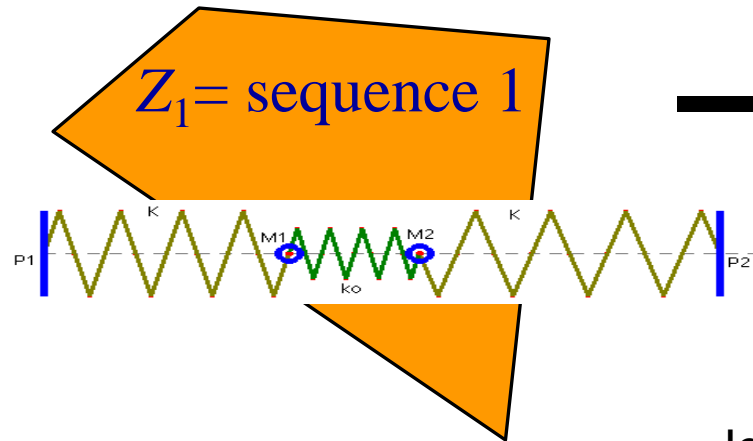
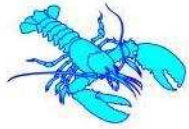
$$x = x^1 + x^2 + \dots + x^n$$

$$z_1 + z_2 + \dots + z_n = 1$$

$$0 \leq z_i \leq 1$$



SHARP  $z_i \in \{0,1\}$  [JEROSLOW84]



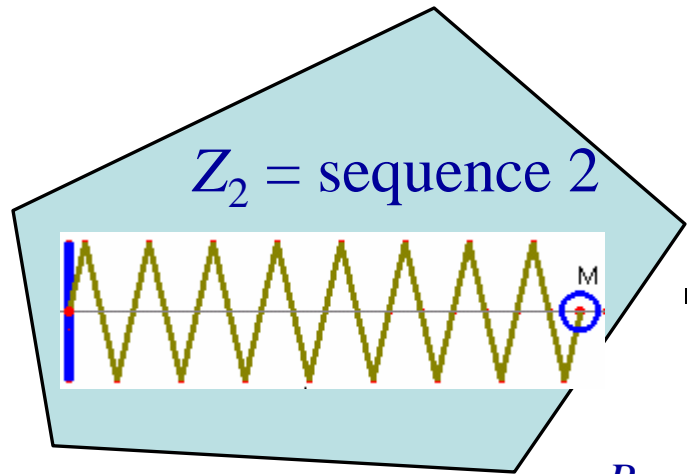
~~$8-M(1-z_1) \leq Begin_1 \leq 10+M(1-z_1)$~~

$8z_1 \leq Begin_1 \leq 10z_1$

$17z_1 \leq End_1 \leq 19z_1$

$7z_1 \leq WorkDuration_1 \leq 8z_1$

Jeroslow sharp formulation is valid since we handle only bounded polyhedrons



$7z_1 \leq Begin_2 \leq 7z_1$

$13z_1 \leq End_2 \leq 14z_1$

$4z_1 \leq WorkDuration_2 \leq 6z_1$

$x = x_1 + x_2 + \dots + x_n$

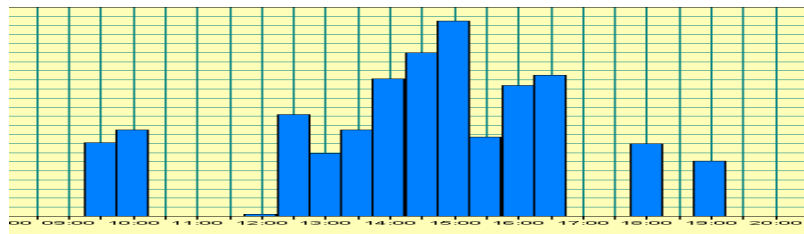
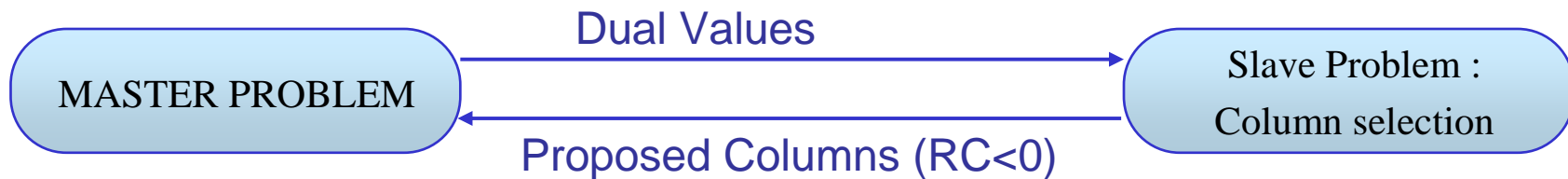
$Begin = Begin_1 + Begin_2$

$End = End_1 + End_2$

$WorkDuration = WorkDuration_1 + WorkDuration_2$



# "WORK" DECOMPOSITION



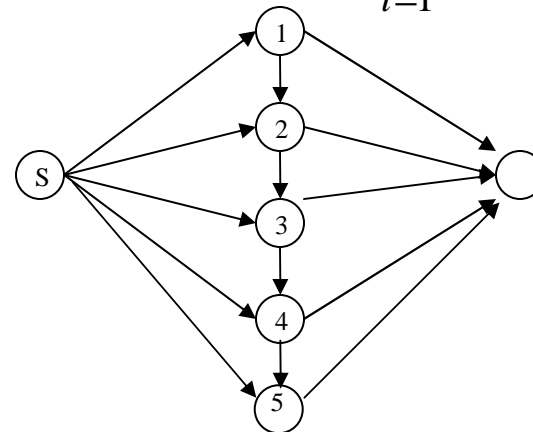
Localized workload++ constraints

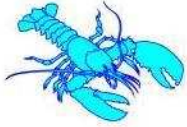
$$\sum_{\text{cols } c \text{ covering } t \text{ by } a} \lambda_c \cdot d_{a,c,t} = sw_{a,t,1} + OverMax_{a,t} + Over_{a,t} - \sum_{i=1}^n Under_{a,t,i}$$

$$Begin_{p,j} = \sum_{\text{cols } c \text{ for } p \text{ in } j} \lambda_c \cdot b_c$$

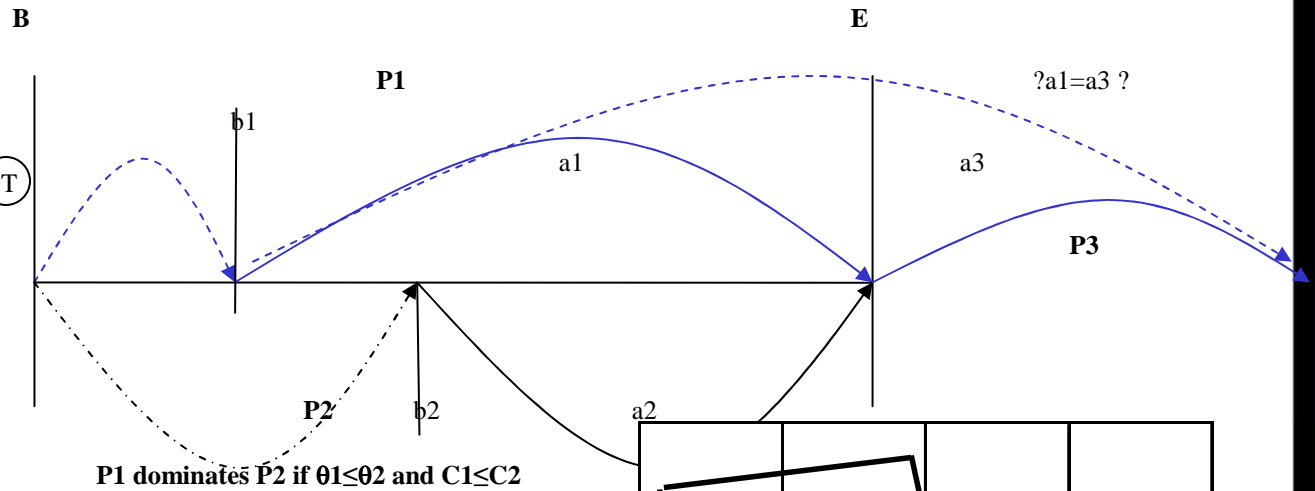
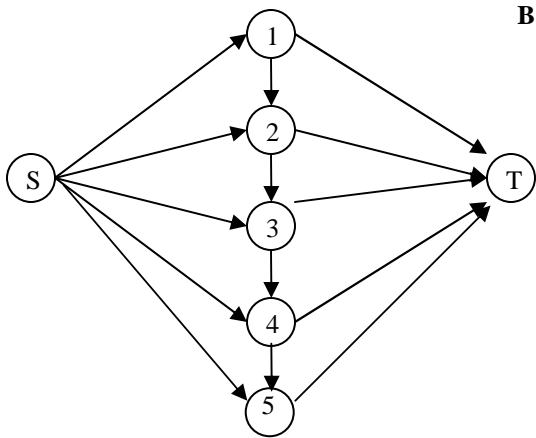
$$End_{p,j} = \sum_{\text{cols } c \text{ for } p \text{ in } j} \lambda_c \cdot e_c$$

$$LateNight_{p,j} = \sum_{\text{cols } c \text{ for } p \text{ in } j} \lambda_c \cdot \nabla_c$$

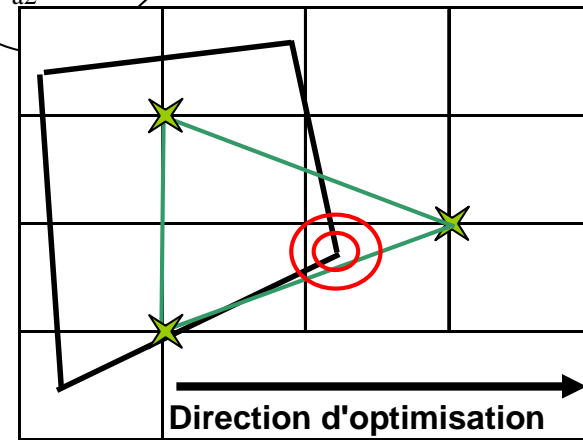




# "WORK BLOCKS" GENERATION



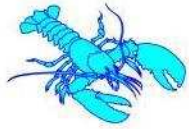
- « Good » WORK BLOCKS are generated using dynamic programming
  - Ressources constraints: number of transitions, activities duration min max
  - Dominacy rules : keep only two best paths



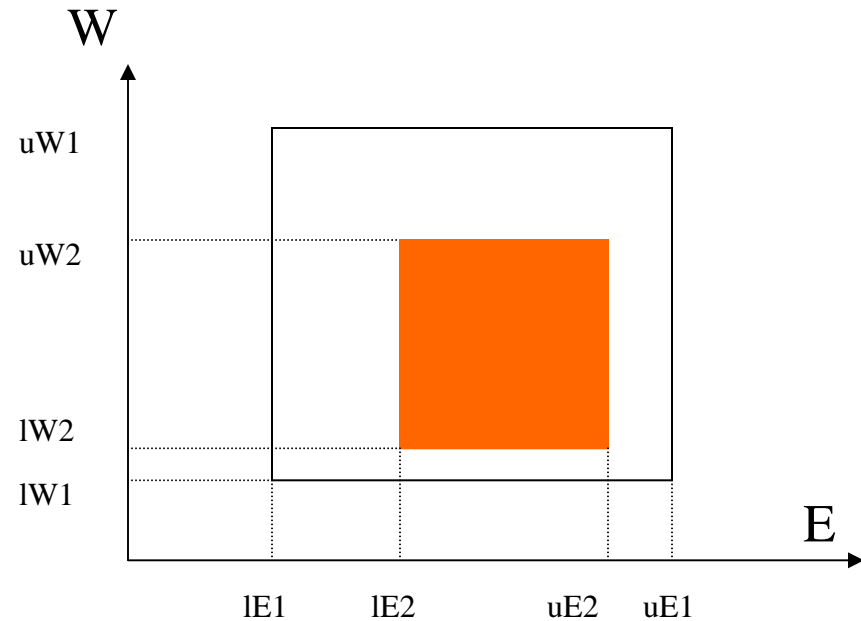
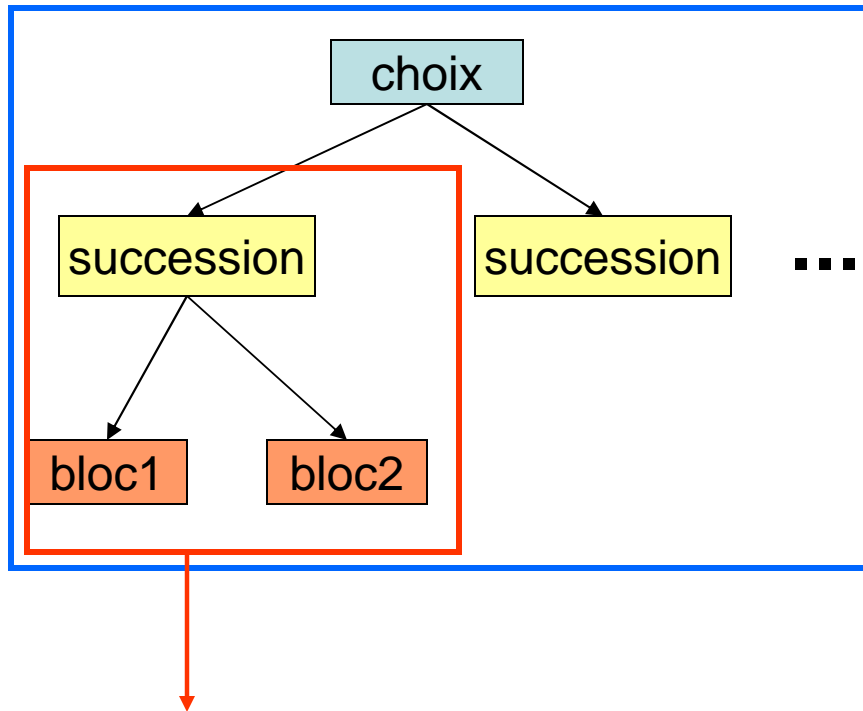
FAST :  $o(\theta^2)$



Continuous masks unfeasible combinations of generated work blocks

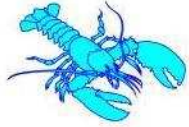


# "DAY" GENERATION



Constraint Programming model : begin, end, length, work duration

- ❖ Enumeration (nested goals)
- ❖ Dominacy
- ❖ Extraction from precomputed "work" best paths



# BRANCH AND PRICE

- continuous process generate too many columns : final MIP is too large, need to control combinatoric=> mix column generation within a global branch & bound
- current node
  - Select branching « master variable » : the one whose value in the continuous simplex is the closest to the equivalent value in a column candidate
  - Select branching direction : anticipation with continuous simplex
  - Branch with removing incompatible previous columns
  - Launch column generation on the local window
  - Loop until there are not ambiguous « master variables » anymore



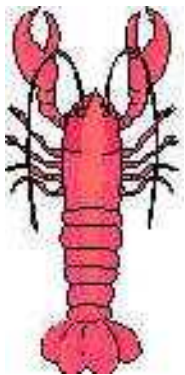
Slow Convergence ,  
needs a new bunch  
of parameters





## Conclusion and next steps

- Generic model works pretty well for both huge flexible teams but also very constrained problems (one or few feasible solutions)
- Importance of master slave frontier
- Needs to convexify the slave model : propagation within branching process could refine dynamically Balas-Jeroslow constraints ([Refalo])!
- Dual pathology : Extremal behavior for mobile activities / cycle dual values for workload... => MASTER needs perturbations



- ❑ ROSTUDEL is looking for customers to adapt this framework, with possible COIN-OR implementation (currently ILOG Cplex / Solver)
- ❑ ROSTUDEL wants to establish links with OR community for current and further issues => cooperation/trainees/job
- ❑ ROSTUDEL expertise on modelization and implementation leads us to establish partnership with other companies